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Testing the gravitomagnetic clock effect on the Earth with neutron interferometry

Lorenzo Iorio*

Lorenzo.Iorio@ba.infn.it

*Dipartimento di Fisica dell' Università di Bari, via Amendola 173, 70126, Bari, Italy

Abstract

The general relativistic gravitomagnetic clock effect consists in the fact that two point particles orbiting a central spinning object along identical, circular equatorial paths, but in opposite directions, exhibit a time difference in describing a full revolution. It turns out that the particle rotating in the same sense of the central body is slower than the particle rotating in the opposite sense. In this paper it is proposed to measure such effect in an Earth laboratory experiment involving interferometry of slow neutrons. With a sphere of 2.5 cm radius and spinning at $4.3 \cdot 10^4$ rad/s as central source, and using neutrons with wavelength of 1 Å it should be possible to obtain, for a given sense of rotation of the central source, a phase shift of 0.18 rad, well within the experimental sensitivity. By reversing the sense of rotation of the central body it should be possible to obtain a 0.06 fringe shift.

1 Introduction

One of the most intriguing consequences of Einstein's General Relativity is the structure of the spacetime around a massive rotating body. The set of its features is named gravitomagnetism [*Ciufolini and Wheeler,* 1995; *Mashhoon et al.,* 2000] since the relativistic equations of motion of a test particle orbiting the central spinning body, in the weak-field and slow-motion approximation, are formally analogous to those of an electrically charged particle acted upon electric and magnetic fields via the Lorentz force.

Some of the consequences of the gravitomagnetic field are:

a) The precession of a gyroscope which will be tested in the field of the Earth by the space mission GP-B [*Everitt et al.,* 1974], scheduled to fly in 2002, at a claimed precision level of 1%

b) The dragging of the local inertial frames which is currently measured in the field of the Earth as well by analyzing the orbits of the laser-ranged passive geodetic satellites LAGEOS and LAGEOS II [*Ciufolini et al.,* 2000]. To date, the accuracy amounts to 20%

c) The gravitomagnetic clock effect [*Mashhoon et al.,* 1999] consisting in the fact that two counter-orbiting test particles placed on two identical, circular equatorial orbits around a central spinning mass take different times to complete a full revolution. It turns out that the particle rotating in the same sense of the central body is slower than the particle moving in the opposite sense. At present, the feasibility of a space-based experiment aimed at the detection of this effect in the terrestrial field is under consideration by the scientific community [*Gronwald et al.,* 1997; *Lichtenegger et al.,* 2000; 2001; *Iorio,* 2000; 2001]

Such relativistic effects are very tiny and their detection in the terrestrial environment via space-based missions is a very demanding task due to many other competing forces acting upon the satellites to be employed which may alias the recovery of the relativistic feature of interest. Last but not least, these kinds of experiments are, obviously, very expensive.

In this paper we will focus on the gravitomagnetic clock effect and on the possibility of measuring it on the Earth in a laboratory experiment involving neutron interferometry [*Rauch and Werner,* 2000]. Such technique has proved itself useful in the so called COW experiments which detected an interference path due to the gravitoelectric field of the Earth [*Overhauser and Colella,* 1974; *Colella et al.,* 1975; *Staudenmann et al.,* 1980; *van der Zouw et al.,* 2000]. In

[*Cohen and Mashhoon,* 1993] it can be found a preliminary discussion of an attempt of detecting the gravitomagnetic field of the Earth in a COW-like interference experiment. A time-shift due to gravitomagnetism in the field of the quantum domain can be found in [*Ahluwalia*, 1997]; see also [*Mashhoon et al.,* 2000; *Mashhoon*, 2000] for the role of gravitomagnetism in quantum theory. For a review of proposed terrestrial experiments aimed at the detection of some other gravitomagnetic features see chapter 6.9 in [*Ciufolini and Wheeler,* 1995].

The paper is organized as follows: in Section 2 we first review a recent proposal for detecting the gravitomagnetic time shift of a couple of electromagnetic orthogonally running waves; subsequently, we describe our approach with neutrons. Section 3 is devoted to a brief discussion on the validity of the approach followed while in Section 4 we summarize the conclusions.

2 Proposal of an actual laboratory experiment

2.1 Electromagnetic interferometry

In [*Tartaglia and Ruggiero,* 2001] the authors work out, in the weak-field and slow-motion approximation, a gravitomagnetic time difference for two electromagnetic waves one running along an equatorial circular path and the other one along a polar one around a central massive spinning body. Subsequently, they propose to detect it in a laboratory experiment on the Earth by using as central source a hollow spherical rotating shell and by means of two fixed orthogonal wave guides in order to make the waves to interfere. The investigated time shift is given by:

$$\Delta T = \frac{\pi}{2}\left(\frac{J}{M}\right)^2 \frac{1}{c^3 R_l} = \frac{2}{9}\frac{\pi}{c^3}\frac{R^2}{R_l}\frac{\sigma_m}{\rho}, \tag{1}$$

in which J is the spin of the central rotating body, M is its mass, c is the speed of light *in vacuo*, R is the radius of the shell, R_l is the radius of the light's path, ρ is the material density of the shell and σ_m is the allowable resistance of the shell's material. Note that such an effect is very tiny because it is of order $\mathcal{O}(c^{-3})$. Indeed, for $\sigma_m = 2000$ MPa, $\rho = 1700$ kg m^{-3} and $R = 1$ m the time shift amounts to $\Delta T = 3 \cdot 10^{-20}$ s only. By using visible light with wavelength of, say, $\lambda = 5 \cdot 10^{-7}$ m it yields a relative phase shift:

$$\Delta\Phi = 2\pi\frac{c}{\lambda}\Delta T \sim 10^{-5}\ rad, \tag{2}$$

with a beam intensity relative change at the interference of only:

$$\delta I/I = \frac{1}{2}(1 - \cos\Delta\Phi) \sim 10^{-9}. \quad (3)$$

2.2 Neutron interferometry

Instead of considering the gravitomagnetic time difference for two electromagnetic waves running along orthogonal paths, for which $\Delta T \sim \mathcal{O}(c^{-3})$ only, we propose to adopt two counter-orbiting, circular beams of slow neutrons running in the equatorial plane of the central body so to adopt the well known [*Mashhoon et al.,* 1999] time shift:

$$\Delta T = T_{+} - T_{-} = 4\pi \frac{J}{Mc^2}, \quad (4)$$

in which T_{+} is the period of the particle rotating in the same sense of the central body and T_{-} is the period of the particle rotating in the opposite sense of the central body: as it can be seen, the co-rotating particle is slower than the counter-rotating one. It is worth noting that eq.(4), which has been derived in the weak-field and slow-motion approximation of General Relativity as well, is of order $\mathcal{O}(c^{-2})$. Moreover, eq.(4), contrary to eq.(1), is independent of the radius of the particles' orbits and depends entirely on the characteristics of the central body. Note also that the time shift amount is independent of the gravitational constant G and of the mass of the central object if it is endowed with suitable symmetries; conversely, it depends linearly on its angular speed Ω and quadratically on its radius R: indeed, for a sphere eq.(4) becomes:

$$\Delta T = 4\pi \frac{I\Omega}{c^2} = \frac{8\pi}{5} \frac{R^2\Omega}{c^2}, \quad (5)$$

where I is the momentum of inertia of the sphere. The maximum angular speed is fixed by the material properties of the object:

$$\Omega_{max} = \sqrt{\frac{\sigma_{max}}{\rho R^2}}, \quad (6)$$

where σ_{max} is the maximum attainable stress at the equator (unidimensional stresses are assumed) and ρ is its mass density [*Tartaglia and Ruggiero,* 2001]. The radius R is limited by the size of the experimental setup: for example, in interferometry experiments with thermal neutrons the typical dimensions of the interferometer are of the order of 10^{-1} m, so that $R = 2.5 \cdot 10^{-2}$ m seems a reasonable value. With such value for the radius and with $\sigma_{max} = 2 \cdot 10^9$

Pa and $\rho = 1700$ Kg m^{-3} [*Tartaglia and Ruggiero,* 2001] the maximum attainable angular speed is $\Omega_{max} = 4.33 \cdot 10^4$ rad s^{-1}. So, by using a small sphere of $2.5 \cdot 10^{-2}$ m radius spinning at $\Omega = 4.33 \cdot 10^4$ rad s^{-1} eq.(4) yields $\Delta T = 1.51 \cdot 10^{-15}$ s which is five orders of magnitude larger than the electromagnetic time shift worked out in [*Tartaglia and Ruggiero,* 2001]. If we use neutrons with De Broglie's wavelength $\lambda_n = h/p_n = h/m_n v_n = 1 \cdot 10^{-10}$ m $\equiv 1$Å, where h is the Planck's constant, we can be successful in raising the relative phase shift to the sensitivity limit of $10^{-2} - 10^{-3}$ rad. Indeed, after a full revolution around the spinning sphere, for a given sense of its rotation, the two neutron beams would accumulate a phase shift at the interference point:

$$\Delta\Phi^{neutron}_{grav} = 2\pi \frac{v_n}{\lambda_n}\Delta T_\pi = 2\pi \frac{m_n v_n^2}{h}\Delta T_\pi = 4\pi^2 \frac{m_n}{hM}(\frac{v_n}{c})^2 J. \tag{7}$$

By considering thermal neutrons with wavelength of 1Å we would have $v_n = 3.9 \cdot 10^3$ m s^{-1} and the phase shift would amount to $\Delta\Phi^{neutron}_{grav} = 1.88 \cdot 10^{-1}$ rad.

Moreover, by repeating the experiment inverting the sense of rotation of the central sphere the neutron beams would experience a time shift $\Delta T^{'} = -\Delta T$ since the beam which was formerly slower now would become faster and vice versa. So it would be possible to observe a fringe shift of:

$$\Delta N = \frac{v_n}{\lambda_n}(\Delta T_\pi - \Delta T^{'}_\pi) = 2\frac{v_n}{\lambda_n}\Delta T_\pi = 6 \cdot 10^{-2} \tag{8}$$

for $\lambda_n = 1 \cdot 10^{-10}$ m. The minimum appreciable fringe shift amounts to 10^{-3}. It is interesting to note that eq.(7) is not independent of the mass of neutron. Moreover, it should be pointed out that in our setup, contrary to the COW experiments, the single crystal interferometer would remain always horizontal, neither it would be rotated along a given axis so that the gravitoelectric field of the Earth would not influence the outcome of the experiment.

Note that by using ultracold neutrons with, e.g., $\lambda_n = 7 \cdot 10^{-8}$ m and $v_n = 5.7$ m s^{-1}, we would obtain, from one hand, larger geometries of the experimental setup would be allowable, but from the other hand the expected phase shift would fall well below the experimental sensitivity which, in this case, is smaller than that of thermal neutrons interferometry.

3 Discussion

Here we will show that we can apply eq.(7) and eq.(4) to our scenario.

Regarding the expression used for the phase shift, let us recall that neutron interferometry, in general, is most appropriately described by taking the Hamiltonian $H = \hat{\mathbf{p}}^2/2m + mU(\mathbf{r})$ and performing a WKB approximation giving a Hamilton-Jacobi equation which can be solved for the modulus of the momentum $p(\mathbf{r}) = p\sqrt{1 + 2m^2 U(\mathbf{r})/p^2}$ where p is the momentum of the neutrons at the beam splitter [*Lämmerzahl,* 1996] and U is the gravitational potential acting upon the particles. In our case $U(\mathbf{r})_{GM} = -c\mathbf{A}_g \cdot \mathbf{v}$ in which $\mathbf{A}_g$ is the gravitomagnetic vector potential given by:

$$\mathbf{A}_g = -\frac{2G}{c^3}\frac{\mathbf{J}\times\mathbf{r}}{r^3}. \tag{9}$$

For paths in the equatorial plane of the central source:

$$U_{GM} = 2\frac{GJ}{c^2}\frac{v}{r^2}. \tag{10}$$

For the geometry of our setup ($J = 1.2$ Kg m^2 s^{-1}, $v_n = 3.95 \cdot 10^3$ m s^{-1}, $r_n = 5 \cdot 10^{-2}$ m) we have: $2m_n^2 U_{GM}/p^2 = 3.59 \cdot 10^{-28}$, so that $p(\mathbf{r}) = p$. In this limit the phase shift, which in general is given by:

$$\Delta\Phi_{grav}^{neutron} = \frac{1}{\hbar}\oint \mathbf{p}(\mathbf{r})\cdot \mathbf{dr}, \tag{11}$$

becomes just eq.(7).

Let us recall that eq.(4) has been worked out in the general relativistic weak-field and slow-motion approximation for a couple of massive particles without internal degrees of freedom following well defined closed geodesic circular paths. Regarding the latter point, because the size of the neutron wave packet can be assumed much more smaller than the macroscopic dimension of the loop formed by the two alternate paths, we can apply the concept of a classical trajectory. Of course, the slow neutrons fly at nonrelativistic speeds and there is no question about the weakness of the gravitomagnetic field generated by the spinning sphere. Concerning the fact that eq.(4) has been derived for geodesic orbits, in our proposed experiment there are no other forces than gravity acting upon the neutrons, apart from the short range nuclear forces at the momentum-conserving reflections [*Lämmerzahl,* 1996] on the crystal mirrors. In fact, in a possible practical realization they should occur only few times, so that, although in an approximate sense, we could assume the neutrons'orbits as geodesic.

It may be useful also to stress that eq.(4) is the time difference as seen by distant inertial observers (or by local non-rotating observers) [*Tartaglia,* 2000]. In our setup the laboratory frame is rotating with the Earth (see [*Tartaglia and Ruggiero,* 2001b] for interesting critical remarks), but, due to the extension of the setup in space and time (the lifetime of neutrons is almost 11 min.) and its geometry, for all practical purpose the adopted frame can be considered as inertial: recall that the proposed angular speed of the central sphere is $\Omega \simeq 10^4$ rad s^{-1}, while the Earth's proper angular speed is $\Omega_\oplus \simeq 10^{-5}$ rad s^{-1}. This suggests that the Sagnac effect should be negligible in this case.

Concerning the fact that neutrons are 1/2 spin particles, under the action of the non-uniform gravitomagnetic field of the rotating body their beams could experience gravitomagnetic Stern-Gerlach forces [*Mashhoon,* 2000]:

$$\mathbf{F}_{SG} = \frac{3GJ}{c^2 r^4}\{[5(\overrightarrow{\sigma}\cdot\hat{\mathbf{r}})(\hat{\mathbf{J}}\cdot\hat{\mathbf{r}}) - \overrightarrow{\sigma}\cdot\hat{\mathbf{J}}] - (\overrightarrow{\sigma}\cdot\hat{\mathbf{r}})\hat{\mathbf{J}} - (\hat{\mathbf{J}}\cdot\hat{\mathbf{r}})\overrightarrow{\sigma}\}, \tag{12}$$

where $\overrightarrow{\sigma}$ is the particle's spin vector ($\sigma = \hbar s$) and r is the particle's position vector. Note that eq.(12) is independent of the neutron mass. For an orbit radius of $r \geq 5\cdot 10^{-2}$ m and for the sphere considered here ($J = 1.2$ Kg m^2 s^{-1}) the amplitude of eq.(12) amounts to $4.4\cdot 10^{-56}$ Kg m s^{-2} only, so that we can completely neglect it and deal with neutrons as test particles.

4 Conclusions

Here we have explored the possibility of setting an Earth laboratory experiment with neutron interferometry aimed at the detection of the phase shift which would be induced on the paths of two coherent beams of slow neutrons by the general relativistic gravitomagnetic field of a small, rapidly spinning sphere. For a given sense of rotation of the latter, the neutrons' phase shift amounts to 0.18 rad. By reversing the sense of rotation of the central source it should be possible to obtain a fringe shift in the interference pattern of 0.06. These results hold for a sphere of 2.5 cm radius spinning at $4.3\cdot 10^4$ rad s^{-1} and for neutrons with a wavelength of 1 Å.

Of course, the practical implications of the realization of the proposed experiment must be analyzed in detail.

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